Open Problems in Heavy Quarkonium Physics

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Abstract. Some recent progress and a personal selection of open problems in heavy quarkonium physics (spectroscopy, decay and production) inspired by the activity of the Quarkonium Working Group are reviewed.

INTRODUCTION

The wealth and quality of new experimental findings and the theoretical progress in the construction and use of Effective Field Theories (EFTs) of QCD are among the reasons of the heavy quarkonium physics *renaissance* witnessed during the last years. In order to keep track and make immediately available to a larger community the progress in the field, experimental and theoretical physicists have gathered in the last three years to form a Quarkonium Working Group [1]. The offspring of this collaboration have been three workshops, a school and a newly issued CERN Yellow Report [2]. In the following we will review some recent results and open problems in heavy quarkonium physics. We refer the reader to [2] for exhaustive presentations.

SPECTROSCOPY

There has been in recent years a renewed interest and a noteworthy progress in the calculation of the heavy quarkonium spectrum from perturbative QCD. The progress comes essentially from three directions:

(1) progress in the construction of EFTs of QCD suitable to describe non-relativistic bound state systems. These have helped to organize higher-order calculations and to factorize high-energy perturbative and low-energy non-perturbative contributions. For a recent review we refer to [3].

(2) fixed order calculations [4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

(3) resummation of large contributions (large logs or large contributions associated to renormalon singularities) [14, 15, 16, 17, 18, 19].

By definition, a perturbative treatment of the bound state is possible if the momentum transfer scale p of the heavy quarks in the bound state is much larger than Λ_{QCD}. Two situations may occur under this condition [20]. Let us call E the typical kinetic energy of the heavy quark and antiquark in the centre-of-mass reference frame: in a non-relativistic bound state $p \sim mv \gg E \sim mv^2$, v being the heavy-quark velocity in the bound state.

The first situation corresponds to quarkonium states for which $E \gtrsim \Lambda_{\mathrm{QCD}}$. Under this circumstance the heavy-quarkonium potential is purely perturbative; we may call this case Coulombic. In the case $E \gg \Lambda_{\mathrm{QCD}}$ non-perturbative contributions are encoded into local condensates, in the case $E \sim \Lambda_{\mathrm{QCD}}$ into non-local ones (see also [21] and references therein). The second situation corresponds to quarkonium states for which $p \gg \Lambda_{\mathrm{QCD}} \gg E$. Under this circumstance the potential contains a perturbative part and short-range non-perturbative contributions. We may call quasi Coulombic the case in which non-perturbative contributions to the potential turn out to be small and may be treated perturbatively in the calculation of observables.

Clearly, perturbative calculations applied to heavy-quarkonium ground states are on a more solid ground that those to higher resonances, and those applied to bottomonium more than those to charmonium, since the (quasi-)Coulombic case is more likely to be realized. Perturbative determinations of the $\Upsilon(1S)$ and J/ψ masses have been used to extract the b and c masses (see *Masses*). The main uncertainty in these determinations comes from non-perturbative contributions. Information on their size can come from other ground-state observables, like the hyperfine splittings (see *Hyperfine splittings*), and the B_c mass (see *B_c mass*). Recent studies of higher quarkonium states in perturbation theory may be found in [4, 22, 23, 5, 24, 25, 12]. Likely only some of the lowest bottomonium resonances may be treated consistently as Coulombic or quasi-Coulombic bound states. It is surprising, therefore, that some of the gross features of the bottomonium spectrum, like the equal spacing of the radial excitations, may be qualitatively reproduced by pure perturbative calculations (see *Higher bottomonium states*).

TABLE 1. Collection of recently obtained values of $m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}})$ and $m_c^{\overline{\mathrm{MS}}}(m_c^{\overline{\mathrm{MS}}})$ from the $\Upsilon(1S)$ and J/Ψ masses.

Ref.	order	$m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}})$ (GeV)
[27]	NNLO	4.24 ± 0.09
[28]	NNLO	4.21 ± 0.09
[29]	NNLO	$4.210 \pm 0.090 \pm 0.025$
[25]	NNLO	$4.190 \pm 0.020 \pm 0.025$
[26]	NNNLO	4.349 ± 0.070
[30]	NNNLO	4.20 ± 0.04
[31]	NNNLO	4.241 ± 0.070

Ref.	order	$m_c^{\overline{\mathrm{MS}}}(m_c^{\overline{\mathrm{MS}}})$ (GeV)
[24]	NNLO	1.24 ± 0.020

Masses

Table 1 shows some recent determinations of the b and c masses from the quarkonium ground-state masses. Finite charm-mass effects have been included in the analyses of [25, 30, 31]. With the exception of [27] the conversion from the threshold (or pole) masses to the $\overline{\mathrm{MS}}$ masses has been performed at three-loop accuracy. All NNNLO analyses are only partially complete, since the full three-loop static potential has not been calculated yet. Different schemes have been used to implement the leading renormalon

cancellation. This may explain some of the differences between the results. The most substantial discrepancy, which is between the result of [26] and all the other ones, may be possibly ascribed to the use of the on-shell scheme, as well as to the way non-perturbative effects have been implemented. In [24] the charm $\overline{\mathrm{MS}}$ mass has been extracted from the J/Ψ mass.

In the above analyses non-perturbative effects constitute the major source of uncertainty together with possible effects due to subleading renormalons. Indeed, in the situation $E \sim \Lambda_{\mathrm{QCD}}$ non-perturbative corrections to the spectrum ($\sim \Lambda_{\mathrm{QCD}}^3/p^2 \sim mv^4$) and the subleading renormalon in the mass ($\sim \Lambda_{\mathrm{QCD}}^2/m \sim mv^4$) are parametrically of the same order as NNLO corrections in the perturbative expansion. As long as non-perturbative effects will not be incorporated in a quantitative manner, perturbative calculations beyond NNLO will not improve the determination of the heavy-quark masses from the quarkonium system.

The dominant non-perturbative correction to the mass of a quarkonium state $|n\rangle$ of LO binding energy $E_n^{(0)}$ is encoded in the expression

$$\delta E_n = -i\frac{g^2}{9}\int_0^\infty dt \, \langle n|\mathbf{r}e^{it(E_n^{(0)}-h_o)}\mathbf{r}|n\rangle \, \langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu),$$

$h_o = p^2/m + \alpha_s/(6r)$ being the LO octet Hamiltonian and μ a factorization scale [8, 20]. A precise determination of this formula would need having an accurate determination of the chromoelectric correlator. In this respect the available data are puzzling. Quenched lattice calculations of gluelump masses indicate an inverse correlation length for the correlator $\langle \mathbf{E}(t)\,\mathbf{E}(0)\rangle$ of about 1.25 GeV [32, 33]. This scale is not only larger than the scale E but also of the same order as the momentum-transfer scale p, if p is identified with $m_b/2 \times 4/3 \times \alpha_s(p)$ (e.g. $m_b \approx 4.7$ GeV and $\alpha_s(p) \approx 0.4$). This may potentially question the whole perturbative approach to the ground state of bottomonium. On the other hand this approach is supported by the fact that it provides a value of the b mass consistent with those obtained from low moments sum rule calculations or from the B system [34]. Only a better determination of the chromoelectric correlator can solve the conundrum. A possible solution could come, for instance, from

- unquenching, if it lowers the value of the inverse correlation length as calculations with cooling techniques seem to indicate [35].
- a particular parametrization of the chromoelectric correlator that sets in the non-perturbative behaviour at a scale μ lower than the inverse correlation length.

The assumption on the Coulombic behaviour of ground-state quarkonia may be also tested on other observables, like the B_c mass and the hyperfine splittings.

B_c mass

Table 2 shows some recent determinations of the B_c mass in perturbation theory at NNLO accuracy compared with the value of the B_c mass in [36] and in a very recent lattice study [37]. The b and c pole masses are expanded in terms of the $\Upsilon(1S)$ and

J/ψ masses respectively in [38], and in terms of the $\overline{\text{MS}}$ masses in [24, 25]. In [25] finite charm-mass effects are included. Since the charm quark effectively decouples in the scheme of Ref. [38], we may consider the results of Refs. [38] and [25] at the same level of theoretical accuracy and the difference between them as due to higher-order corrections beyond NNLO accuracy.

TABLE 2. Different perturbative determinations of the B_c mass compared with the experimental value and a recent lattice determination.

B_c mass (MeV)					
State	[36] (expt)	[37] (lattice)	[38] (NNLO)	[24] (NNLO)	[25] (NNLO)
1^1S_0	6400(400)	$6304 \pm 12^{+12}_{-0}$	6326(29)	6324(22)	6307(17)

In *Fermilab Today*, December 3, 2004, the CDF collaboration has announced the preliminary results of a search for the B_c, using decays into a J/ψ and a charged pion. Their preliminary result is $M_{B_c} = 6287 \pm 5$ MeV. This value, if confirmed, would support the assumption that non-perturbative contributions to the quarkonium ground state are of the same magnitude as NNLO or even NNNLO corrections, which would be consistent with a $E \gtrsim \Lambda_{\text{QCD}}$ power counting.

Hyperfine splittings

Charmonium and bottomonium ground state hyperfine splittings have been recently calculated at NLL in [18]. The result in the charmonium case is consistent with the experimental value. This, again, supports the assumption that non-perturbative contributions for the quarkonium ground state are consistent with a $E \gtrsim \Lambda_{\text{QCD}}$ power counting. In the bottomonium case there are not yet experimental data.

We note, however, that NNLO corrections may be potentially important for this observable, also considering that at this order new type of corrections will appear for the first time:

- non-perturbative corrections, which are parametrically of NNLO in the situation $\Lambda_{\text{QCD}} \sim E$.
- corrections to the mass. In order to show the possible impact of this type of corrections, in Fig. 1 we display the hyperfine splittings at NLO accuracy calculated using $1/2$ of the bottomonium and charmonium vector ground state masses and the $\overline{\text{MS}}$ masses. The difference between the two determinations (taken around the maximum in μ) is about 70% in the charmonium case and about 30% in the bottomonium case.

Higher bottomonium states

Higher bottomonium resonances have been investigated in the framework of perturbative QCD most recently in [24, 25, 39]. The calculation of [24] is accurate at NNLO, that one of [25] includes finite charm-mass effects, while [39] is a numerical calculation
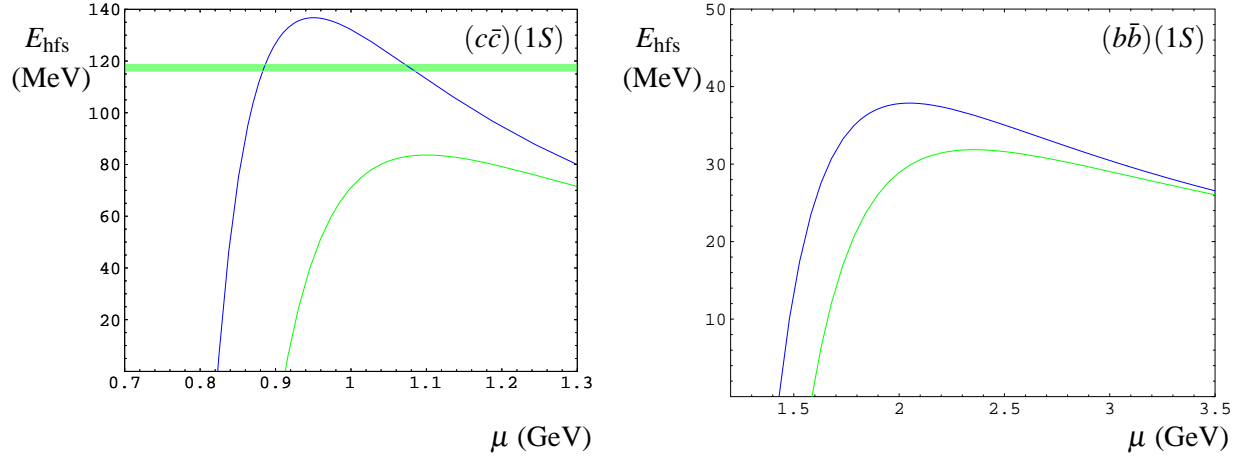
FIGURE 1. Hyperfine splittings of the $1S$ charmonium (left) and bottomonium (right) states at NLO in the $1S$ mass scheme, $m_c = M_{J/\psi}/2$, $m_b = M_{\Upsilon(1S)}/2$, (lower curves) and in the $\overline{\text{MS}}$ mass scheme, $m_c = m_c^{\overline{\text{MS}}}(m_c^{\overline{\text{MS}}})(1 + 4\alpha_s/(3\pi))$, $m_b = m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}})(1 + 4\alpha_s/(3\pi))$, (upper curves) as a function of the normalization scale μ. α_s runs at 4-loop accuracy with 3 massless flavours. Left plot: $m_c^{\overline{\text{MS}}}(m_c^{\overline{\text{MS}}}) = 1.28$ GeV, $\alpha_s(M_{J/\psi}/2) = 0.352$, $\alpha_s(m_c^{\overline{\text{MS}}}(m_c^{\overline{\text{MS}}})) = 0.405$. Right plot: $m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}}) = 4.22$ GeV, $\alpha_s(M_{\Upsilon(1S)}/2) = 0.209$, $\alpha_s(m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}})) = 0.218$.

TABLE 3. Masses of $b\bar{b}$ states. The result of Ref. [24] comes from a full perturbative calculation up to $O(m\alpha_s^4)$ without finite charm-mass corrections; the result of Ref. [25] comes from a full perturbative calculation up to $O(m\alpha_s^4)$ including finite charm-mass corrections; the result of [39] incorporates full corrections up to $O(m\alpha_s^4)$ in the individual levels and full corrections up to $O(m\alpha_s^5)$ in the fine splittings, includes finite charm-mass corrections and also depends on some assumptions about the long-range non-perturbative behaviour of the static potential. Numbers without errors are those without explicit or reliable error estimates in the corresponding works.

$b\bar{b}$ states				
State	expt	[24]	[25]	[39]
1^3S_1	9460	9460	9460	9460
1^3P_2	9913	9916(59)	10012(89)	9956
1^3P_1	9893	9904(67)	10004(86)	9938
1^3P_0	9860	9905(56)	9995(83)	9915
2^3S_2	10023	9966(68)	10084(102)	10032
2^3P_2	10269		10578(258)	10270
2^3P_1	10255		10564(247)	10260
2^3P_0	10232	10268	10548(239)	10246
3^3S_1	10355	10327(208)	10645(298)	10315

of the spectrum that includes also NLO spin-dependent potentials. This last calculation also relies on some assumptions about the long-range behaviour of the static potential. The results are summarized in Table 3.

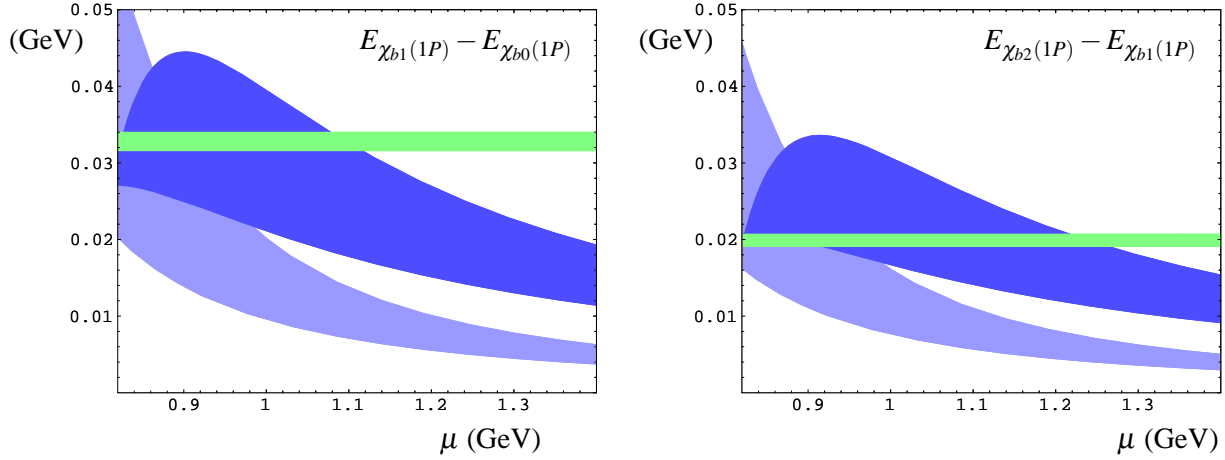The surprising result of these studies is that some gross features of the lowest part of

FIGURE 2. $E_{\chi_{b1}(1P)} - E_{\chi_{b0}(1P)}$ (left) and $E_{\chi_{b2}(1P)} - E_{\chi_{b1}(1P)}$ (right) versus the normalization scale μ. The light band shows the LO expectation, the dark one the NLO one. The widths of the bands account for the uncertainty in $\alpha_s(M_Z) = 0.1187 \pm 0.002$ [36]. The horizontal bands show the experimental values [36]. From [13].

the bottomonium spectrum, like the approximate equal spacing of the radial levels, is reproduced by a perturbative calculation that implements the leading-order renormalon cancellation. If this is coincidental or reflects the (quasi-)Coulombic nature of the states will be decided by further studies. A recent NLO calculation of the $1P$ bottomonium fine splittings has been performed in [13]. The results are summarized in Fig. 2. Figure 3 plots ρ, the ratio of the fine splittings considered in Fig. 2, as a function of the normalization scale μ (see [40] and references therein for early studies of this quantity). It seems to indicate either the existence of large NLL/NNLO corrections (as it happens in the hyperfine splittings of the $1S$ levels) or sizeable non-perturbative corrections, somehow hidden in the error bands of Fig. 2.
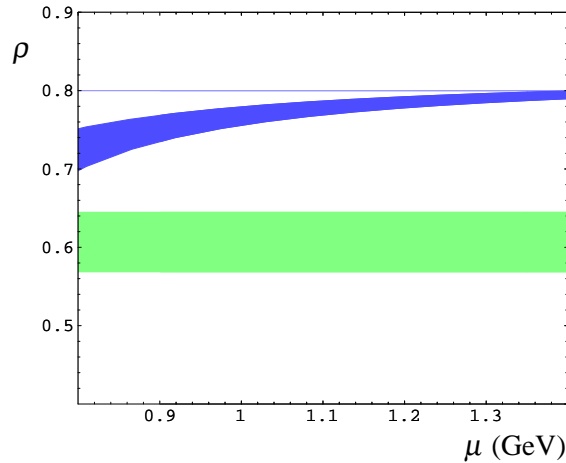

FIGURE 3. $\rho = (E_{\chi_{b2}} - E_{\chi_{b1}})/(E_{\chi_{b1}} - E_{\chi_{b0}})$ versus the normalization scale μ. The horizontal line at 0.8 corresponds to the LO expectation, the curve to the NLO one. The band accounts for the uncertainty in $\alpha_s(M_Z) = 0.1187 \pm 0.002$ [36]. The horizontal band shows the experimental value [36]. From [13].
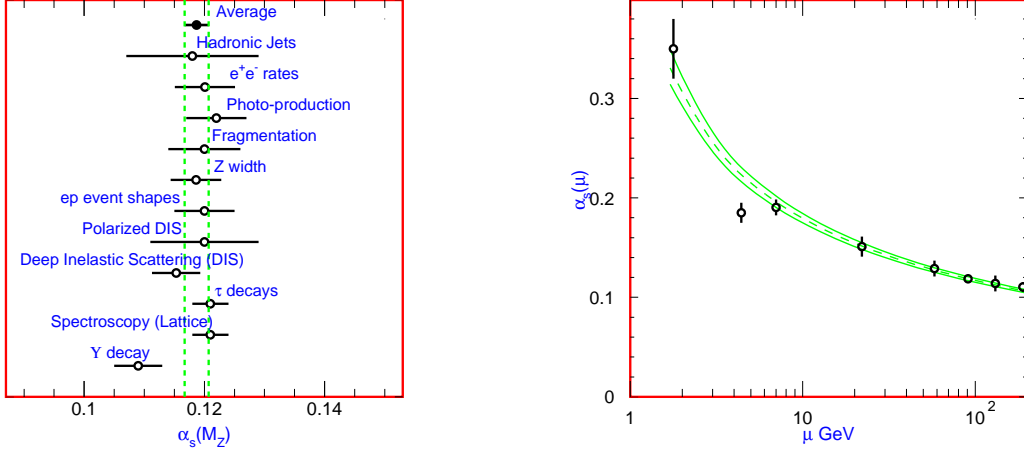
FIGURE 4. Different determinations of $\alpha_s(M_Z)$ (left) and of $\alpha_s(\mu)$ at different scales μ (right). In both figures the determinations outside the band come from Υ decay data. From [36].

DECAY

Υ inclusive decays

The NRQCD factorization formulas state that inclusive decay widths to light hadrons (*l.h.*) may be written as sums of products of matrix elements and imaginary parts of Wilson coefficients of 4-fermion operators [41]. In the case of the Υ system they read:

$$\Gamma(\Upsilon \to l.h.) = \frac{2}{m^2} \left(\operatorname{Im} f_1(^3S_1) \langle O_1(^3S_1) \rangle_\Upsilon \right.$$

$$+ \operatorname{Im} f_8(^3S_1) \langle O_8(^1S_0) \rangle_{\eta_b} + \frac{\operatorname{Im} f_8(^1S_0)}{3} \langle O_8(^3S_1) \rangle_{\eta_b}$$

$$\left. + \operatorname{Im} g_1(^3S_1) \frac{\langle \mathscr{P}_1(^1S_0) \rangle_{\eta_b}}{m^2} + \frac{\sum_J (2J+1)\operatorname{Im} f_8(^3P_J)}{9} \frac{\langle O_8(^1P_1) \rangle_{\eta_b}}{m^2} + \cdots \right).$$

The use of these formulas (and similar ones for the electromagnetic decay widths) to extract α_s at the b-mass scale has turned out problematic, as shown by Fig. 4. It seems that uncertainties have been underestimated [2]. The reasons may be the following:

- poor knowledge of the matrix elements. Note that usually octet matrix elements have been neglected in the analyses. This may not be justified considering that $\operatorname{Im} f_1(^3S_1)$ and $\operatorname{Im} g_1(^3S_1)$ are suppressed by an extra α_s with respect to most of the $\operatorname{Im} f_8$ coefficients (see [42] and references therein) that may compensate, under some circumstances, the suppression in the velocity expansion of the octet operator matrix elements. The poor knowledge of the matrix element $\langle \mathscr{P}_1 \rangle$ has been indicated as one of the major source of uncertainty in [43]. Studies of decay

matrix elements on the lattice can be found in [44, 45, 46]. Factorization formulas for decay matrix elements in pNRQCD have been derived in [47, 48].

• large higher-order corrections to the Wilson coefficients [49].

TABLE 4. Comparison of ratios of χ_{cJ} partial widths. The experimental values PDG 2004 and PDG 2000 are obtained from the world averages of [36] and [50] respectively. The chosen ratios do not depend at leading order in the velocity expansion on octet or singlet matrix elements. The LO and NLO columns refer to a leading and next-to-leading order calculation done at the renormalization scale $2m_c$ with the following choice of parameters: $m_c = 1.5$ GeV and $\alpha_s(2m_c) = 0.245$. From [2].

Ratio	[36] (PDG 2004)	[50] (PDG 2000)	LO	NLO
$\dfrac{\Gamma(\chi_{c0} \to \gamma\gamma)}{\Gamma(\chi_{c2} \to \gamma\gamma)}$	5.1 ± 1.1	13 ± 10	≈ 3.75	≈ 5.43
$\dfrac{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c0} \to \gamma\gamma)}$	410 ± 100	270 ± 200	≈ 347	≈ 383
$\dfrac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c0} \to \gamma\gamma)}$	3600 ± 700	3500 ± 2500	≈ 1300	≈ 2781
$\dfrac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c2} \to l.h.)}{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}$	7.9 ± 1.5	12.1 ± 3.2	≈ 2.75	≈ 6.63
$\dfrac{\Gamma(\chi_{c0} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}{\Gamma(\chi_{c2} \to l.h.) - \Gamma(\chi_{c1} \to l.h.)}$	8.9 ± 1.1	13.1 ± 3.3	≈ 3.75	≈ 7.63

χ_c inclusive decays

In the case of χ_c inclusive decay widths the NRQCD factorization formulas read:

$$\Gamma(\chi_{cJ} \to l.h.) = 9 \operatorname{Im} f_1(^3P_J) \frac{\left|R'_{\chi_{cJ}}(0)\right|^2}{\pi m^4} + \frac{2 \operatorname{Im} f_8(^3S_1)}{m^2} \langle O_8(^1S_0)\rangle_{\chi_{cJ}} + \cdots,$$

$$\Gamma(\chi_{cJ} \to \gamma\gamma) = 9 \operatorname{Im} f_{\gamma\gamma}(^3P_J) \frac{\left|R'_{\chi_{cJ}}(0)\right|^2}{\pi m^4} + \cdots \qquad J = 0, 2,$$

where $R'_{\chi_{cJ}}(0)$ is the derivative of the radial part of the χ_{cJ} wave function at the origin. The displayed terms are accurate at leading order in the velocity expansion. The Wilson coefficients are known at NLO accuracy in α_s.

The experimental determination of the χ_c decay widths has dramatically improved in the last four years mainly due to the measurements of the E835 experiment at the Fermilab Antiproton Accumulator. A way to see the impact of these measurements is provided by Tab. 4, where we compare the PDG 2000 [50] with the PDG 2004 [36] determinations of different ratios of hadronic and electromagnetic widths. There have been sizable shifts in some central values and considerable reductions in the errors. In particular, the error on the ratio of the electromagnetic χ_{c0} and χ_{c2} widths has been reduced by about a factor 10, while in all other ratios the errors have been reduced by a factor 2 or 3. The considered ratios of hadronic and electromagnetic widths do not depend at leading order in the velocity expansion on any non-perturbative parameter. Therefore, they can be determined in perturbation theory only. The last two columns of Tab. 4 show the result of a LO and NLO calculation respectively. Clearly the data have become sensitive to NLO corrections and may be used, in principle, to determine α_s at the charm-mass scale. Before this, a necessary step is the calculation of the decay widths at next-to-leading order in the velocity expansion, since these contributions are potentially of the same magnitude as NLO corrections in the Wilson coefficients (see Ref. [51] for a calculation in the electromagnetic case).

PRODUCTION

In this section we briefly mention two of the main open problems in our understanding of charmonium production. We refer to [2] for a proper treatment.

Charmonium polarization

Quarkonium production at large p_T is dominated by gluon fragmentation. NRQCD predicts that the dominant gluon-fragmentation process is gluon fragmentation into a quark-antiquark pair in a color octet $^3S_1$ state. At large p_T the fragmenting gluon is transversely polarized. In the standard NRQCD power counting it is expected that the octet quark-antiquark pair keeps the transverse polarization as it evolves into a S-wave quarkonium. Different power countings [52] or higher-order corrections may somehow dilute the polarization, which, however, is expected to show up in high p_T data. The present Tevatron data [53] do not seem to confirm this expectation. The uncertainties are, however, too large to make any definite claim. The issue here is mainly experimental and more precise Tevatron data are eagerly awaited.

Double charmonium production

The most challenging open problem in charmonium production concerns double charmonium production in e^+e^- annihilations. The Belle data of inclusive $J/\psi + c\bar{c}$

production,

$$\frac{\sigma(e^+e^- \to J/\psi + c\bar{c})}{\sigma(e^+e^- \to J/\psi + X)} = 0.82 \pm 0.15 \pm 0.14, \quad \text{Belle [54]},$$

and of exclusive $J/\psi + \eta_c$ production,

$$\sigma(e^+e^- \to J/\psi + \eta_c)\,\text{Br}(c\bar{c}_{\text{res}} \to > 2 \text{ ch. track}) = 25.6 \pm 2.8 \pm 3.4 \text{ fb}, \quad \text{Belle [55]},$$

are far by almost an order of magnitude from the available theoretical predictions:

$$\frac{\sigma(e^+e^- \to J/\psi + c\bar{c})}{\sigma(e^+e^- \to J/\psi + X)} \approx 0.1, \quad [56, 57, 58],$$

$$\sigma(e^+e^- \to J/\psi + \eta_c) = 2.31 \pm 1.09 \text{ fb}, \quad [59, 60].$$

On the other hand, the present upper bound of Belle on $J/\psi + J/\psi$ production is consistent with theoretical expectations:

$$\sigma(e^+e^- \to J/\psi + J/\psi)\,\text{Br}(c\bar{c}_{\text{res}} \to > 2 \text{ ch. track}) < 9.1 \text{ fb}, \quad \text{Belle [55]},$$

$$\sigma(e^+e^- \to J/\psi + J/\psi) = 8.70 \pm 2.94 \text{ fb}, \quad [61, 62].$$

Even if independent data by BaBar would be most welcome, the issue here seems to be mainly theoretical and the data may be the signal of some new production mechanism.

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